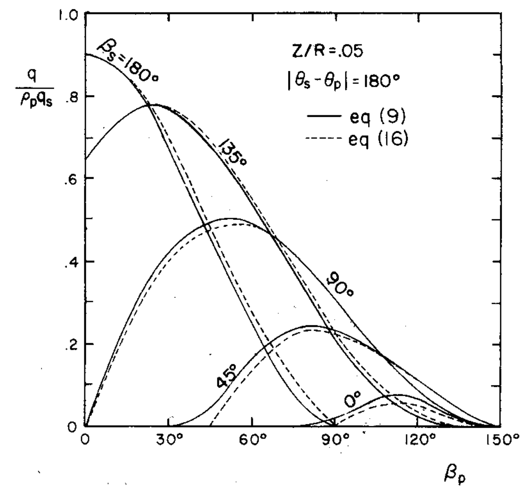

Fig. 4 Solar albedo flux as function of surface-planet tilt angle β_p.

It is seen that whenever $\mathfrak{F}$ is appreciable the approximate result yields excellent accuracy. The error is never larger than 0.05. Only when $\mathfrak{F}$ is very small does the relative error become large: for $\beta_s \leq 90$ deg Eq. (17) predicts $\mathfrak{F} \leq 0$, and $\mathfrak{F}$ should be set equal to zero.

In the following figures $Z = 0.05\ R$ was chosen as a typical orbit. Figure 3 shows the influence of solar position β_s and θ_s. It is seen that the reflected solar flux is a maximum near the direction where $\cos\bar{\alpha}$ is a maximum, i.e., at

$$\tan\beta_s = \tan\beta_p \cos(\theta_p - \theta_s) \tag{19}$$

This is the reason why the approximation, Eq. (17), is so successful as is seen again in this figure. (For clarity the approximate solution is not shown for all cases.)

A final comparison is shown in Fig. 4 for sun and planet on opposing sides, $|\theta_p - \theta_s| = 180$ deg with sun polar angle β_s as parameter. The approximate maxima are again given by Eq. (19), i.e., $\beta_s + \beta_p = 180$ deg. As before, the approximation gives excellent results for all cases.

The simple approximate formula displays excellent accuracy for all significant situations, with an error which is always less than 5% of the maximum possible reflected flux. It is felt that this approximation is adequate for all practical applications as the common and concurrent assumption of a gray and diffuse planet results in errors of at least the same magnitude.

Acknowledgment

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References

[1] Cunningham, F. G., "Power Input to a Small Flat Plate from a Diffusely Radiating Sphere, With Application to Earth Satellites," NASA TN D-710, 1961.

[2] Siegel, R. and Howell, J. R., *Thermal Radiation Heat Transfer*, McGraw-Hill, New York, 1972.

[3] Modest, M. F., "Computer Code MCDREF-User's Manual (Monte Carlo Determination of Radiative Exchange Factors)," Crew Systems Division, NASA-JSC, Contract No. NAS9-15109, Houston, Texas, 1979.

[4] Modest, M. F. and Poon, S. C., "Determination of Three-Dimensional Radiative Exchange Factors for the Space Shuttle by Monte Carlo," ASME Paper 77-HT-49, 1977.

[5] Modest, M. F., "Three-Dimensional Radiative Exchange Factors for Nongray, Nondiffuse Surfaces," *Numerical Heat Transfer*, Vol. 1, 1978, pp. 403-416.

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Emittance of a Finite Scattering Medium with Refractive Index Greater Than Unity

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Introduction

REFRACTIVE index and scattering can significantly influence the transfer or radiation in a semitransparent medium such as water, glass, plastics, or ceramics. In a recent article,[1] the author presented exact numerical results for the emittance of a semi-infinite scattering medium with a refractive index greater than unity. The objective of the present investigation is to extend the analysis to a finite medium.

Formulation

The physical situation consists of a finite planar layer. The isothermal layer emits, absorbs, and isotropically scatters thermal radiation. It is characterized by a single scattering albedo ω, an optical thickness τ_0, a refractive index n, and a temperature T. The interface at $\tau = 0$ is assumed to be smooth and its reflection and transmission characteristics are governed by Snell's law and the Fresnel relations. The bottom ($\tau = \tau_0$) is transparent. The intensity incident on the interface from inside the medium is given by[1]

$$I^-(0,\mu) = n^2 I_b(T)\epsilon_m(\mu) + 2\int_0^1 \rho(\mu')I^-(0,\mu')\rho_m(\mu,\mu')\mu'\,d\mu' \tag{1}$$

where μ is the cosine of the polar angle inside the medium, $I_b(T)$ is the Planck function, and $\rho(\mu)$ is the interface reflectance. Physically, $\epsilon_m(\mu)$ and $\rho_m(\mu,\mu_0)$ are the directional emittance and the bidirectional reflectance, respectively, of a finite medium with a refractive index of unity. These properties are simply related to Chandrasekhar's X and Y functions[2,3]:

$$\rho_m(\mu,\mu_0) = (\omega/4)\,[X(\mu;\tau_0)X(\mu_0;\tau_0) - Y(\mu;\tau_0)Y(\mu_0;\tau_0)]/(\mu+\mu_0) \tag{2}$$

$$\epsilon_m(\mu) = \left(1 - \frac{\omega}{2}\alpha_0 - \frac{\omega}{2}\beta_0\right)[X(\mu;\tau_0) - Y(\mu;\tau_0)] \tag{3}$$

where the moments are defined as

$$\alpha_0 = \int_0^1 X(\mu;\tau_0)\,d\mu \text{ and } \beta_0 = \int_0^1 Y(\mu;\tau_0)\,d\mu.$$

Since integral Eq. (1) is linear, $I^-(0,\mu)$ can be expressed in terms of a universal function $f(\mu)$, i.e., $I^-(0,\mu) = n^2 I_b(T)$

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$f(\mu)$ where

$$f(\mu)=\epsilon_m(\mu)+2\int_0^1 \rho(\mu')f(\mu')\rho_m(\mu,\mu')\mu' \mathrm{d}\mu' \qquad (4)$$

The directional emittance at $\mu_1=\cos\theta_1$, as measured on the outside of the interface, is defined by

$$\epsilon(\mu_1)=[1-\rho(\mu)]I^-(0,\mu)/[n^2 I_b(T)]=[1-\rho(\mu)]f(\mu) \qquad (5)$$

where $\mu=[1-(1-\mu_1^2)/n^2]^{1/2}$ represents the direction in which the intensity within the medium must hit the interface in order to appear at the vacuum side in the direction μ_1 (Snell's law).

Results

Integral Eq. (4) is solved by successive approximations. The X and Y functions are calculated from the following two integro-differential equations:

$$X'=Y\Phi, \quad Y'=-(Y/\mu)+X\Phi \qquad (6)$$

where

$$\Phi=(\omega/2)\int_0^1 Y(\mu';\tau_0)(\mathrm{d}\mu'/\mu')$$

with initial conditions, $\tau_0=0, X=Y=1$. The prime indicates the derivative with respect to optical thickness, τ_0. The hemispherical emittance is calculated from

$$\epsilon_H=2\int_0^1 \epsilon(\mu_1)\mu_1 \mathrm{d}\mu_1$$

For a nonscattering medium ($\omega=0$), the directional emittance is given by

$$\epsilon(\mu_1)=[1-\rho(\mu)][1-\exp(-\tau_0/\mu)] \qquad (7)$$

The directional emittance is just the emittance of the "medium" multiplied by the transmittance of the interface. The ratio of the directional emittance to the normal emittance is presented in Fig. 1. The emittançe is zero at $\mu_1=0$, increases, reaches a maximum, and then decreases with μ_1. As the optical thickness increases, the angle θ_1 for maximum emittance decreases. This maximum is most pronounced at small refractive indices.

The directional character of the emittance can be approximated by the nonscattering result, i.e.

$$[\epsilon(\mu_1)/\epsilon_N]_\omega \simeq [\epsilon(\mu_1)/\epsilon_N]_{\omega=0}$$

$$=\frac{(n+1)^2[1-\rho(\mu)][1-\exp(-\tau_0/\mu)]}{4n[1-\exp(-\tau_0)]} \qquad (8)$$

Table 1 Emittance of a finite medium

τ_0	ω=0	ω=0.10	ω=0.50	ω=0.90	ω=0.95	ω=0.99	ω=0	ω=0.10	ω=0.50	ω=0.90	ω=0.95	ω=0.99
	Normal Emittance for n = 1.5						Normal Emittance for n = 2.0					
0.1	0.09136	0.08432	0.05216	0.01177	0.00598	0.001212	0.08459	0.07812	0.04847	0.01098	0.00558	0.001132
0.2	0.17402	0.16270	0.10703	0.02624	0.01350	0.002765	0.16113	0.15083	0.09980	0.02468	0.01271	0.002606
0.3	0.24881	0.23476	0.16171	0.04257	0.02216	0.004583	0.23038	0.21771	0.15122	0.04036	0.02106	0.004364
0.4	0.31649	0.30069	0.21490	0.06026	0.03173	0.006626	0.29305	0.27895	0.20148	0.05760	0.03043	0.006377
0.5	0.37773	0.36686	0.26590	0.07897	0.04204	0.008865	0.34975	0.33486	0.24983	0.07607	0.04069	0.008621
0.6	0.43314	0.41568	0.31433	0.09841	0.05295	0.01128	0.40106	0.38581	0.29585	0.09549	0.05172	0.01108
0.7	0.48328	0.46557	0.35999	0.11835	0.06435	0.01384	0.44748	0.43218	0.33933	0.11563	0.06339	0.01374
0.8	0.52864	0.51093	0.40283	0.13859	0.07614	0.01653	0.48949	0.47435	0.38017	0.13627	0.07561	0.01658
0.9	0.56969	0.55215	0.44287	0.15897	0.08823	0.01935	0.52749	0.51267	0.41837	0.15723	0.08830	0.01959
1.0	0.60684	0.58959	0.48017	0.17936	0.10055	0.02227	0.56188	0.54747	0.45398	0.17835	0.10136	0.02276
1.5	0.74580	0.73094	0.62974	0.27808	0.16355	0.03807	0.69055	0.67882	0.59668	0.28226	0.16986	0.04056
2.0	0.83008	0.81775	0.73000	0.36645	0.22525	0.05513	0.76859	0.75946	0.69208	0.37656	0.23886	0.06077
2.5	0.88120	0.87094	0.79597	0.44178	0.28276	0.07277	0.81592	0.80883	0.75465	0.45715	0.30409	0.08243
3.0	0.91220	0.90346	0.83890	0.50412	0.33474	0.09054	0.84463	0.83902	0.79525	0.52365	0.36339	0.10486
3.5	0.93101	0.92333	0.86664	0.55469	0.38075	0.10817	0.86205	0.85745	0.82141	0.57734	0.41591	0.12754
4.0	0.94242	0.93545	0.88446	0.59517	0.42089	0.12544	0.87261	0.86870	0.83820	0.62007	0.46161	0.15008
5.0	0.95353	0.94736	0.90314	0.65248	0.48511	0.15841	0.88290	0.87974	0.85576	0.68013	0.53432	0.19372
6.0	0.95762	0.95178	0.91070	0.68760	0.53152	0.18880	0.88669	0.88384	0.86287	0.71665	0.58640	0.23435
7.0	0.95912	0.95343	0.91374	0.70881	0.56442	0.21634	0.88808	0.88536	0.86571	0.73858	0.62305	0.27130
8.0	0.95968	0.95403	0.91496	0.72150	0.58747	0.24098	0.88859	0.88592	0.86685	0.75167	0.64856	0.30436
9.0	0.95988	0.95426	0.91544	0.72907	0.60349	0.26282	0.88878	0.88613	0.86730	0.75945	0.66622	0.33357
10.0	0.95996	0.95434	0.91563	0.73356	0.61456	0.28201	0.88885	0.88621	0.86748	0.76407	0.67838	0.35915
∞	0.96000	0.95439	0.91575	0.74008	0.63888	0.39802	0.88889	0.88626	0.86760	0.77076	0.70498	0.51047
	Ratio of Hemispherical to Normal Emittance for n = 1.5						Ratio of Hemispherical to Normal Emittance for n = 2.0					
0.1	1.0689	1.0689	1.0689	1.0690	1.0690	1.0690	1.0053	1.0053	1.0053	1.0054	1.0054	1.0054
0.5	1.0418	1.0419	1.0424	1.0430	1.0430	1.0431	0.9929	0.9929	0.9932	0.9934	0.9935	0.9935
1.0	1.0150	1.0152	1.0164	1.0180	1.0182	1.0184	0.9801	0.9803	0.9809	0.9818	0.9819	0.9820
1.5	0.9947	0.9950	0.9965	0.9990	0.9994	0.9998	0.9702	0.9704	0.9713	0.9728	0.9730	0.9732
2.0	0.9799	0.9801	0.9816	0.9844	0.9850	0.9855	0.9627	0.9629	0.9639	0.9658	0.9662	0.9665
2.5	0.9692	0.9693	0.9705	0.9732	0.9738	0.9744	0.9572	0.9573	0.9583	0.9604	0.9609	0.9613
3.0	0.9616	0.9617	0.9624	0.9646	0.9652	0.9658	0.9532	0.9533	0.9541	0.9562	0.9572	0.9572
4.0	0.9529	0.9528	0.9524	0.9527	0.9531	0.9535	0.9483	0.9484	0.9488	0.9503	0.9509	0.9515
5.0	0.9489	0.9487	0.9475	0.9456	0.9455	0.9456	0.9461	0.9461	0.9460	0.9468	0.9472	0.9478
6.0	0.9472	0.9469	0.9452	0.9414	0.9407	0.9403	0.9451	0.9450	0.9447	0.9447	0.9449	0.9453
8.0	0.9462	0.9459	0.9437	0.9374	0.9356	0.9340	0.9445	0.9443	0.9438	0.9426	0.9424	0.9425
10.0	0.9461	0.9457	0.9434	0.9359	0.9334	0.9305	0.9443	0.9443	0.9437	0.9419	0.9413	0.9410
∞	0.9461	0.9457	0.9434	0.9352	0.9316	0.9247	0.9443	0.9442	0.9436	0.9415	0.9404	0.9385

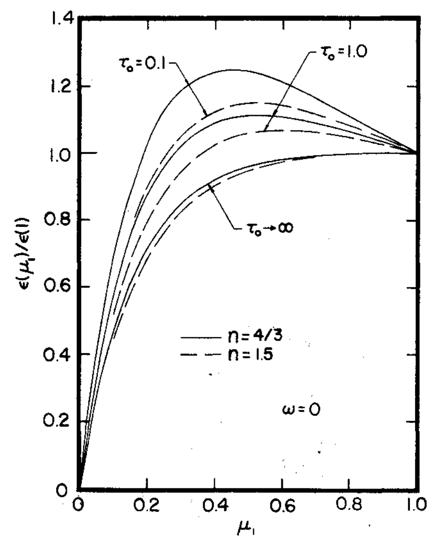

Fig. 1 Directional behavior of the emittance for a nonscattering medium.

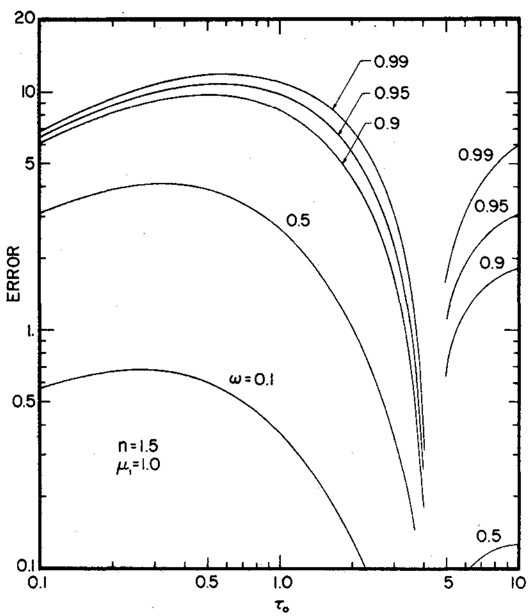

Fig. 2 Percentage error for Eq. (9) with $\mu_1 = 1.0$ and $n = 1.5$.

The largest error in this approximation is at large τ_0 and ω and small n and μ_1. For $\omega \leq 0.99$, the largest error is 10, 6, and 2%, respectively, for $n=4/3$, 1.5, and 2.0. It has been suggested[4] that the directional character of the emittance is almost independent (within 10%) of refractive index, scattering albedo, and optical thickness. Inspection of Fig. 1 reveals that this observation is incorrect, except possibly for large optical thicknesses.

Numerical results for the normal emittance is presented in Table 1 for $n=1.5$ and 2.0. Inspection of Table 1 reveals that the normal emittance is decreased by scattering ($\omega>0$). An increase in refractive index decreases the emittance for a nonscattering medium ($\omega=0$); however, for a scattering medium the emittance first increases, reaches a maximum, and then decreases with refractive index. As the albedo is increased, the refractive index at which the maximum occurs also increases. This maximum is shifted to larger refractive indices when the albedo or optical thickness is increased. As might be expected, an increase in optical thickness is always associated with an increase in emittance. The influence of refractive index is most pronounced at large optical thicknesses, i.e., a semi-infinite medium.

The ratio of hemispherical to normal emittance is presented in Table 1. The ratio is greater than unity for small optical thicknesses and less than unity for large optical thicknesses. This behavior is because the directional emittance is greater than normal emittance for most directions at small optical thicknesses.

A close inspection of the numerical results reveals that $f(\mu)$ or $I^-(0,\mu)$ is a weak function of μ. This is especially true for $1 \leq \mu \leq \mu_c$. Employing the first term of Taylor expansion of $f(\mu')$ about $\mu'=\mu$, i.e., $f(\mu')=f(\mu)$, in Eq. (4) yields the following approximation:

$$f_a(\mu)=\epsilon_m(\mu)/[1-2\int_0^1 \rho(\mu')\rho_m(\mu,\mu')\mu' d\mu'] \tag{9}$$

The validity of this approximation is illustrated in Fig. 2 for $\mu_1=1$ and $n=1.5$. The largest error occurs for normal emittance and small amounts of absorption ($\omega \simeq 1$). For $\tau_0<5$, the approximation of Eq. (9) underestimates the emittance, while for $\tau_0>5$ the emittance is overestimated. Thus, Eq. (9) is accurate to within 15% for $\omega \leq 0.99$ and $4/3 \leq n \leq 2$.

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References

[1] Crosbie, A.L., "Emittance of a Semi-Infinite Scattering Medium with Refractive Index Greater than Unity," *AIAA Journal,* Vol. 17, Jan. 1979, pp. 117-120.

[2] Crosbie, A.L., "Emittance of an Isothermal, Isotropically Scattering Medium," *AIAA Journal,* Vol. 11, Aug. 1973, pp. 1203-1205.

[3] Crosbie, A.L., "Reflection Function for an Isotropically Scattering Finite Medium," *AIAA Journal,* Vol. 11, Oct. 1973, pp. 1448-1450.

[4] Armaly, B.F., "Emittance of an Isothermal, Isotropically Scattering Finite Medium: Influence of Refractive Index," AIAA Paper 77-743, 1977.